



Fast Route **Detailed Route**

Collective motion





Fig. 1: (a) Starlings flocking and (b) fish schooling.

Often we need only look out of the window to see the principles of collective behaviour at work. Behaviours which capture our imaginations are bird flocks, fish schools, locust plagues and bee swarms. As well as the aesthetically pleasing aspect of watching a swarm in motion, studying collective behaviour has practical applications: understanding fish schooling can lead to more well developed fishing strategies; a knowledge of the way locusts interact and stay together in devastatingly large, coherent groups may shed light on possible strategies which may be used to disrupt these groups and halt the swarm's destructive progress [1,2]. This last scenario is of particular interest for this work.

Experimental setup

Groups of (between 5 and 100) locust nymphs were placed in a ring-shaped arena (see Fig 2 (a)) and their movements recorded [3]. Low density (~20 individuals) groups aligned and marched in one direction around the ring for up to 2 or 3 hours, before spontaneously switching direction in the space of only a few minutes.







Fig. 2 (b): Average velocity, U, of 30 locusts over 8 hours.

The group property of average velocity, U, characterises the locusts switching behaviour. $|U| \sim 1$ indicates the locusts are aligned and marching in the same direction. Fig. 2 (b) gives an example of spontaneous switching.

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Individual-based model

to the behaviour of neighbours found less than a distance R (the interaction radius) from it. Locust *i*'s behaviour is described by its position, $x_i \equiv x_i(t)$, and velocity, $u_i \equiv u_i(t)$, i = 1, ..., N. On a one-dimensional domain (with periodic boundary conditions) the position and velocity evolve according to:

$$\Delta x_i = u_i \Delta t, \qquad (1)$$

$$\Delta u_i = \left\{ G\left(\bar{u}_i^{loc}\right) - u_i(t) \right\} \Delta t + \Delta Q \eta \left(\bar{u}_i^{loc}\right), \qquad (2)$$

where Δt is the time step between successive position and velocity updates. ΔQ is a random variable uniformly distributed in $\left[-\sqrt{\Delta t}\,\omega/2,\sqrt{\Delta t}\,\omega/2\right]$, where ω is the (constant) system noise amplitude. \bar{u}_i^{loc} is the mean of the velocities of the particles local to (inside interaction radius, R, of) particle i and the function $G: \mathbb{R} \to \mathbb{R}$ is chosen to be



Fig. 3: Average velocity, U, of 30 simulated locusts looks qualitatively similar to the experimental switching behaviour.

Effective SDE for average velocity

Qualitatively, the above model reproduces the switching behaviour seen in the data (compare Fig. 2 (b) and Fig. 3). Switching behaviour of this type can also be reproduced using a Stochastic Differential Equation (SDE) describing a diffusion process in a two-welled potential:

$$dU = F(U)dt + \sqrt{2D(U)}dW(t),$$

By considering lots of small bursts of simulation or small data

where F(U) is the drift coefficient, D(U) is the diffusion coefficient and dW(t) is the standard Wiener process. time-courses we extract the coefficients of the coarse-grained SDE numerically [1,4]. Initialising the model with a desired value of U and running the simulation for a short time, δt , we can find the coefficients using

$$F(U) \approx \left\langle \frac{U(t+\delta t) - U(t)}{\delta t} \right\rangle, \qquad (4)$$
$$D(U) \approx \frac{1}{2} \left\langle \frac{[U(t+\delta t) - U(t)]^2}{\delta t} \right\rangle. \qquad (5)$$



This is me. I look a bit like this. If I'm not standing here and you want to speak to me about this work then please come and find me at some point during the meeting and I'll be happy to talk to you about it. Or you can email me at yatesc@maths.ox.ac.uk or visit my website

http://people.maths.ox.ac.uk/yatesc/ for more information.

United by noise: Randomness helps swarms stay together

Consider N 'locusts'. Each locust adjusts its behaviour according

$$f(z) = \frac{1}{1+\beta} \{ z + \beta \operatorname{sign}(z) \}$$

where β is a positive constant and $\eta\left(\bar{u}_{i}^{loc}\right) \equiv 1$ initially, so the local average velocity measured does not influence the size of the noise.

(3)

Refining the model

The drift coefficients for the model and the experimental data (see Fig. 4 (a) and (b) respectively) compare well qualitatively. The diffusion coefficients, however, differ significantly. For the experimental data the diffusion coefficient appears to have a quadratic shape, with its maximum at zero average velocity.

The increase in the diffusion coefficient when group alignment is low indicates that the locusts might respond to low group alignment by increasing the noisiness of their motion. To test this hypothesis we refined the model. Instead of taking the function η (multiplying the uniform random variable, ΔQ , in the velocity update Eq. (2)) to be unity, we chose it to be a nontrivial function of the local mean velocity, \bar{u}_i^{loc} specifically:

$$\eta\left(\bar{u}_{i}^{loc}\right) = \frac{3}{2} \left\{ 1 - \left(\frac{\bar{u}_{i}^{loc}}{|\bar{u}_{i}^{loc}|_{max}}\right)^{2} \right\},\tag{6}$$

where $|\bar{u}_i^{loc}|_{max}$ is the maximum of the absolute value of the mean local velocity. In Fig. 5 (c) the quadratic nature of the effective diffusion coefficient displayed in the experimental data (see Fig. 5 (b)) is recovered by the refined model.



Fig. 4: Drift coefficients of underlying SDEs for (a) the original individualbased model (b) experimental data and (c) the refined individual-based model. All are roughly cubic with varying degrees of noise.





Fig. 5: Diffusion coefficients of underlying SDEs for (a) the original individual-based model (b) the experimental data and (c) the refined individual-based model. The quadratic shape of the experimental diffusion coefficient is matched qualitatively by the diffusion coefficient of the refined model but not by the diffusion coefficient of the original model.

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Conclusions

We used our experimental observations to guide our modelling, changing the noise term for individual locusts so that it becomes larger when its perceived local alignment is smaller. This allowed us to match the diffusion coefficients of SDEs assumed to underlie the data and the model more accurately.

This result indicates that the individuals move more randomly in locust groups with low alignment. This appears to enable the group to find (and remain in) a highly aligned state more easily. This may have important implications regarding attempts to control the motion of locust swarms.



for why individual locusts might increase their randomness when they find themselves in an unaligned state.

Recent findings about cannibalistic interactions between marching locusts [5] may provide a rationalization for the observation of apparently increased individual randomness in response to a loss of alignment at the group level: given the risk of exposing the rear of the abdomen to oncoming insects [5], there may be selection pressure on an individual to minimize the time spent in the disordered phase.

Reference

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