

## Timetable

	<b>Tuesday 6</b>		<b>Wednesday 7</b>	<b>Thursday 8</b>	<b>Friday 9</b>
09:00-10:00	<i>Registration</i>	09:00-10:00	Plenary Lecture	<b>Dominique Bakry</b>	Plenary Lecture
10:30-11:30	Opening Session	10:00-10:30	<i>Coffee Break</i>	<i>Coffee Break</i>	<i>Coffee Break</i>
		10:30-11:15	<b>Patricia Reynaud-Bouret</b>	<b>Luis Bergues</b>	<b>Diogo Arsénio</b>
		11:15-12:00	<b>Anna Marciniak</b>	<b>Joaquín Fontbona</b>	<b>Hisao Fujita</b>
		12:00-12:45	<b>Pierre Gabriel</b>	<b>Carlos Mora</b>	<b>Nicolas Meunier</b>
		12:45-13:30	<i>Lunch</i>	<i>Lunch</i>	<b>Marzia Bisi</b>
		13:00-14:00	<i>Lunch</i>	<i>Lunch</i>	Plenary Lecture
14:00-15:00	Plenary Lecture	14:00-15:00	Plenary Lecture	Plenary Lecture	<i>Closing Address</i>
15:00-15:30	<i>Tour</i>	15:15-16:00	<b>Nathalie Krell</b>	<b>Karl-Theodor Sturm</b>	
15:30-16:00	<b>Mariano Rodríguez</b>	16:10-16:30	<i>Coffee Break</i>	<i>Coffee Break</i>	
16:00-16:30	<b>Alexander Lorz</b>	16:30-17:15	<b>Mar González</b>	<b>Dario Cordero</b>	
16:30-17:00	<b>Erika Carretto</b>	17:15-18:00	<b>Ivan Gentil</b>	<b>Pietro Caputo</b>	
17:00-17:30	<b>Ernesto Nungesser</b>				
19:30-23:00	<i>Welcome Party</i>				

## Diogo Arsénio

### *From the Vlasov-Maxwell-Boltzmann system to incompressible viscous electro-magneto-hydrodynamics*

Under suitable hydrodynamic regimes, the Vlasov-Maxwell-Boltzmann system converges, at least formally, towards an incompressible Navier-Stokes-Fourier system coupled with self-induced electromagnetic forces. In this talk, we discuss the rigorous justification of this asymptotic regime in the framework of renormalized solutions, both with and without cutoff assumptions. We will put special emphasis on the new mathematical difficulties specific to the Vlasov-Maxwell-Boltzmann system. In particular, we will discuss the persistence of acoustic and electromagnetic waves, the hypoellipticity in kinetic transport equations, and the existence of measure-valued renormalized solutions and of renormalized solutions with a defect measure.

This is a joint work with Laure Saint-Raymond.

## Dominique Bakry

### *Diffusions and orthogonal polynomials*

Diffusion semigroups are described through their generators, which are in general in  $\mathbb{R}^n$  or an open set in it second order differential operators of the form

$$L(f)(x) = \sum_{ij} a^{ij}(x) \frac{\partial^2 f}{\partial x_i \partial x_j} + \sum_i b^i(x) \frac{\partial f}{\partial x_i}$$

The easiest cases are when one is able to diagonalize this operator in an basis of orthogonal polynomials, since then one is able to have a quite explicit description of the associated law of the underlying process. In dimension 1, there are not many examples of such a situation. It reduces to the family of Jacobi, Laguerre and Hermite polynomials. In higher dimension, many examples come from Lie group actions of homogeneous spaces, or generalizations of them, through root systems or other algebraic constructions. We shall give a complete characterization of the problem : on which open sets in  $\mathbb{R}^n$  one may expect to find a probability measure for which the associated orthogonal polynomials are eigenvectors of diffusion operators. We shall give a complete description of all the models in dimension 2, where we are able to completely solve this problem. There are exactly 11 compact sets (up to affine transformations), and 7 non compact ones, on which there exist such a measure. We shall also describe all the associated measures and operators.

## Marzia Bisi

### *Multi-temperature hydrodynamic equations from kinetic theory for rarefied gas mixtures*

Starting from the Boltzmann kinetic equations for a mixture of gas molecules, we show that the asymptotic limit assuming mechanical encounters between particles of the same species dominant (fast processes) with respect to all other (mechanical or chemical) interactions, leads to multi-temperature and multi-velocity hydrodynamic equations. Preliminary results have been obtained for a mixture of four gases with only translational degrees of freedom, subject also to a bimolecular and reversible chemical reaction. More desirable is the generalization of this procedure to a mixture of gas molecules whose internal structure is described by a discrete set of internal energy levels. Even in this case the Euler equations for densities, velocities and temperatures of each component are derived. Balance equations retain collision terms, contributed by the slow processes, which have to be closed, in the zero order Chapman-Enskog expansion, by the fast collision equilibrium. Analytical expressions for such contributions may be achieved in special situations, like Maxwell collision model, or hard-sphere interactions and only one energy level per species.

## Luis Bergues

La electroterapia en el cáncer viene cobrando auge en el tratamiento de tumores; sin embargo, no está implementada en la Oncología clínica, a pesar de sus prometedores en humanos, porque la misma no se ha estandarizado ni se conoce el mecanismo de acción. La no estandarización se explica en parte porque no se conoce la dosis óptima por tipo de tumor y los modelos existentes no explican las diferentes respuestas de los tumores después de aplicada la terapia, como progresión de la enfermedad, enfermedad estable, respuesta parcial y remisión completa. El objetivo de la presentación es la modificación de la ecuación de Gompertz para describir dichas respuestas. Un nuevo tipo de respuesta fue revelada, la respuesta parcial estacionaria. Se concluye que la ecuación de Gompertz modificada es factible para describir las cinéticas de crecimiento de los tumores no perturbados y perturbados con corriente eléctrica directa.

### **Erika Carretto**

*An estimate of the effect of nonlinear term in the stochastic Navier-Stokes equation characterizing the "energy cascade" in a turbulent flow*

In this paper, drawing inspiration from the theory of Kolmogorov and Obukhov of 1941, we study the problem of turbulence through the stochastic Navier-Stokes equations. In particular, we consider the stochastic Navier-Stokes equations on the bi-dimensional torus  $\mathbb{T}^2$  and its invariant measure. The choice of  $\mathbb{T}^2$  allows us to make use of Fourier series. To study turbulence, we examine relations between the local average  $\bar{v}$  of velocity  $v$  and the fluctuation  $u = v - \bar{v}$ . And for this purpose, we introduce a family of local average operators parametrized by the characteristic width  $\delta > 0$ , more precisely the convolution with a function

$$\Theta_\delta = \mathcal{F}^{-1} \left( \frac{1}{1 + \delta |\xi|^2} \right).$$

The main result of this paper is the following theorem.

**THEOREM** *We suppose that  $v$  is a solution of the Navier-Stokes equations*

$$dv = [-\mathcal{P}_{\mathcal{H}}(v \cdot \nabla)v + \nu \Delta v] dt + dW,$$

*realizing an invariant measure. We set*

$$\bar{v} = \Theta_\delta * v.$$

*Then there exists a function  $f(\delta)$ ,  $\delta > 0$ , such that*

$$f(\delta) \rightarrow 0 \quad \text{for} \quad \delta \rightarrow 0$$

*and that*

$$\begin{aligned} \nu \mathbb{E} \|\nabla \bar{v}\|_{L^2(\mathbb{T}^2)}^2 + N_\delta &= \frac{1}{2} \sum_{(j,k) \in \mathbb{L}} \frac{1}{(1 + \delta |k|^2)^2} \lambda_{j,k}^2, \\ |N_\delta| &\leq C f(\delta) \sum_{(j,k) \in \mathbb{L}} \lambda_{j,k}^2 \left( \sum_{(j,k) \in \mathbb{L}} |k|^2 \lambda_{j,k}^2 \right)^{\frac{1}{2}} \end{aligned}$$

This theorem gives an estimate of the eventual energy "cascade" from motion of big length to motions of small length. This estimate of the energy transfer depends on  $\delta$ , in such a way that it tends to 0 when  $\delta$  tends to 0.

### **Pietro Caputo**

*On the relaxation to equilibrium of random surfaces*

We consider the stochastic evolution of a class of random surfaces naturally arising in combinatorial structures - dimer coverings of the honeycomb lattice - and in statistical mechanics - Ising interfaces and

the Solid-On-Solid model. Under the assumption of planar boundary conditions, we establish mixing time behavior which agrees with the diffusive scaling prediction, with bounds that are tight up to logarithmic corrections. The analysis rests on some new coupling ideas which allow us to show that the relaxation pattern roughly follows a deterministic mean curvature motion.

Joint work with F. Martinelli and F.L. Toninelli.

### **Joaquín Fontbona**

#### *A trajectorial interpretation of entropy dissipation and a non intrinsic Bakry-Emery criterion*

We develop a pathwise description of the dissipation of general convex entropies for continuous time Markov processes, based on simple backward martingales and convergence theorems with respect to the tail sigma field. The entropy is in this setting the expected value of a backward submartingale. In the case of (non necessarily reversible) Markov diffusion processes, we use Girsanov theory to explicit its Doob-Meyer decomposition, thereby providing a stochastic analogue of the well known entropy dissipation formula, valid for general convex entropies (including total variation). Under additional regularity assumptions, and using It calculus and some ideas of Arnold, Carlen and Ju, we obtain a new Bakry Emery criterion which ensures exponential convergence of the entropy to 0. This criterion is non-intrinsic since it depends on the square root of the diffusion matrix, and cannot be written only in terms of the diffusion matrix itself. We provide an example where the classic Bakry Emery criterion fails, but our non-intrinsic criterion ensuring exponential convergence to equilibrium applies without modifying the law of the diffusion process.

Joint work with Benjamin Jourdain.

### **Pierre Gabriel**

#### *Optimal growth for linear processes with affine control - Application to a protein amplification technique*

We consider the controlled dynamical system  $\dot{t} = (G + \alpha F)t$ , where  $G$  and  $F$  are given matrices with nonnegative extra-diagonal terms. We show the existence of an optimal Perron eigenvalue with respect to parameter  $\alpha$  under some assumptions. Then we prove the existence of an eigenvalue (in the generalized sense) for the full optimal control problem when  $\alpha = \alpha(t)$  is a time dependent control. Surprisingly enough, the two eigenvalues appear to be numerically the same.

This is a joint work with Vincent Calvez.

### **Ivan Gentil**

#### *Logarithmic Sobolev inequality applied to non-linear Cauchy problems*

In a forth coming preprint we prove the existence of a weak solution of a general reaction-diffusion equation. The logarithmic Sobolev inequality is one of the main tool of this work.

### **María del Mar González**

#### *Classical solutions for a nonlinear Fokker-Planck equation arising in Computational Neuroscience*

We analyze the global existence of classical solutions to the initial boundary value problem for a nonlinear parabolic equation describing the collective behavior of an ensemble of neurons. These equations were obtained as a diffusive approximation of the mean-field limit of a stochastic differential equation system. The resulting Fokker-Planck equation presents a nonlinearity in the coefficients depending on the probability flux through the boundary. We show by an appropriate change of variables that this parabolic equation with nonlinear boundary conditions can be transformed into a non standard Stefan-like free boundary problem with a source term given by a delta function. We prove that there are global classical

solutions for inhibitory neural networks, while for excitatory networks we give local well-posedness of classical solutions together with a blow up criterium. Finally, we will also study the spectrum for the linear problem corresponding to uncoupled networks and its relation to Poincare inequalities for studying their asymptotic behavior.

**Hisao Fujita Yashima**

*Mathematical modelling of the motion of atmosphere with phase transition of water*

We propose a mathematical model of the motion of atmosphere, which takes into account the phase transition of water in the atmosphere, that is, formation and evaporation of clouds, rain and snow. The physical quantities which we consider in this equation system are the density of dry air  $\rho$ , the density of vapour  $\pi$ , the density of liquid water  $\sigma_l(m)$  contained in the drops of mass  $m$ , the density of solidified water  $\sigma_s(m)$  contained in small ice bodies of mass  $m$ , the velocity of the air  $v = (v_1, v_2, v_3)$ , the velocity of the water drops of mass  $m$ ,  $u_l(m) = (u_{l,1}(m), u_{l,2}(m), u_{l,3}(m))$ , the velocity of the small ice bodies of mass  $m$ ,  $u_s(m) = (u_{s,1}(m), u_{s,2}(m), u_{s,3}(m))$ , the temperature  $T$  and the pressure  $p$ . This equation system contains some parabolic equations (essentially for  $v$  and  $T$ ) and some transport equations (essentially for  $\rho$ ,  $\pi$ ,  $\sigma_l$  and  $\sigma_s$ ), while  $p$ ,  $u_l(m)$  and  $u_s(m)$  are considered as function of other quantities ( $\rho$ ,  $T$ ,  $v$ ,  $m$ ). We prove the existence and the uniqueness of the local solution to a slightly modified equation system. The method of the proof is based on a standard fix point argument, but the estimation of  $\sigma_l(m)$  and  $\sigma_s(m)$  for linearised equation is delicate. In order to remove the unnatural condition of this our result, we consider the stationary solution to the transport equation for  $\sigma_l$ , reducing it to a Smoluchowski-type equation under the gravitation. We prove the existence and the uniqueness of the stationary solution under a horizontal wind. This technique can be used also for the global solution. Another important aspect of the atmosphere physics is the effect of the radiation. We consider the equation of the radiation and of its thermic effects and we prove the existence of a stationary solution. As far as the numeric methods, we develop a simulation of the wind which goes over the mountains. As the coefficients on the terms of first derivatives are much greater than those on the terms of second derivatives, we use the finite difference method. The diminution of the temperature on the mountains obtained in this simulation coincides very well with what the physical theory predicts.

**Nathalie Krell**

*Statistical inference for structured populations alimented by transport-fragmentation*

We investigate inference in simple models that describe the evolution in size of a population of bacteria across scales. The size of the system evolves according to a transport-fragmentation equation: each individual grows with a given transport rate, and splits into two offsprings, according to a binary fragmentation process with unknown division rate that depends on its size. Macroscopically, the system is well approximated by a PDE and statistical inference transfers into a nonlinear inverse problem. Microscopically, a more accurate description is given by a stochastic piecewise deterministic Markov process, which allows for other methods of inference, introducing however stochastic dependences. We will discuss and present some very simple results on the inference of the parameters of the system across scales. Real data analysis is conducted on E. Coli experiments.

This is a joint on-going work with M. Doumic (INRIA and Paris 6), M. Hoffmann (ENSAE) and L. Robert (INSERM).

**Alexander Lorz**

*Dirac Mass Dynamics in Parabolic Equations*

Nonlocal Lotka-Volterra models have the property that solutions concentrate as Dirac masses in the limit of small diffusion. Is it possible to describe the dynamics of the concentration points and of the mass of the Dirac? We will explain how this relates to the so-called 'constrained Hamilton-Jacobi equation'

and how numerical simulations can exhibit unexpected dynamics well explained by this equation. Our motivation comes from 'populational adaptive evolution' a branch of mathematical ecology which models the darwinian evolution.

Authors

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**Anna Marciniak-Czochra**

*Spike patterns in reaction-diffusion-ode systems*

In this talk we explore a mechanism of pattern formation arising in the processes described by a system of a single reaction-diffusion equation coupled with ordinary differential equations. Such systems of equations arise for example from the modelling of the interactions between cellular processes and diffusing growth factors. We focus on the models of early cancerogenesis proposed by Marciniak-Czochra and Kimmel, but the theory we develop applies to the wider class of pattern formation models with an autocatalytic non-diffusing component.

Such models can exhibit diffusion-driven instability (Turing-type instability). However, they are very different from classical Turing-type models and the spatial structure of the pattern emerging from the destabilisation of the spatially homogeneous steady state cannot be concluded from a linear stability analysis. The models exhibit qualitatively new patterns of behaviour of solutions, including a strong dependence of the emerging pattern on initial conditions and quasi-stability followed by rapid growth of solutions. We prove that, under very general assumptions on nonlinearities, all Turing-type patterns, *i.e.*, regular stationary solutions, are unstable in the Lyapunov sense. In numerical simulations, solutions having the form of periodic or irregular spikes are observed.

The talk is based on a joint research with Kanako Suzuki (Tohoku University), Grzegorz Karch (University of Wroclaw) and Steffen Harting (University of Heidelberg)

**Nicolas Meunier**

*Study of mathematical models for atherosclerosis*

In this talk we will study different mathematical models for phenomena that are involved in atherosclerosis.

**Carlos Mora**

*Stochastic Schrodinger equations with unbounded coefficients*

We will focus on stochastic partial differential equations of Schrodinger type. These stochastic evolution equations describe the dynamics of quantum systems interacting with heat baths. The talk will address basic properties of the linear and non-linear stochastic Schrodinger equations, like the well-posedness of solutions and existence of invariant measures (see, e.g., [1, 2, 3]). Moreover, we will discuss the relation between the stochastic Schrodinger equations and the operator equations describing the evolution of quantum observables (see, e.g., [4]). Using the connection between the stochastic Schrodinger equations and the quantum master equations (see, e.g., [5]) we will obtain the existence of regular stationary solutions for the quantum master equations.

**Ernesto Nungesser**

*Future asymptotics of homogeneous cosmological models*

The late-time behaviour of the Einstein-Euler system with Bianchi symmetry is well understood. Sometimes it is of advantage to use kinetic theory to describe the matter content of the universe. In particular, collisionless matter is often used in astrophysics and has some nice mathematical properties. We will present some results concerning the late-time behaviour of the Einstein-Vlasov system with Bianchi A symmetry. The results imply in particular that collisionless matter is well approximated by the Einstein-dust system.

**Patricia Reynaud-Bouret**

*Nonparametric estimation of the division rate of a size-structured population*

We consider the problem of estimating the division rate of a size-structured population in a nonparametric setting. The size of the system evolves according to a transport-fragmentation equation: each individual grows with a given transport rate, and splits into two offsprings of the same size, following a binary fragmentation process with unknown division rate that depends on its size. In contrast to a deterministic inverse problem approach, as in (Perthame, Zubelli, 2007) and (Doumic, Perthame, Zubelli, 2009), we take in this paper the perspective of statistical inference: our data consists in a large sample of the size of individuals, when the evolution of the system is close to its time-asymptotic behavior, so that it can be related to the eigenproblem of the considered transport-fragmentation equation. By estimating statistically each term of the eigenvalue problem and by suitably inverting a certain linear operator (see previously quoted articles), we are able to construct a more realistic estimator of the division rate that achieves the same optimal error bound as in related deterministic inverse problems. Our procedure relies on kernel methods with automatic bandwidth selection. It is inspired by model selection and recent results of Goldenschluger and Lepski.

This is a joint work with M. Doumic-Jauffret, M. Hoffmann and V. Rivoirard

**Mariano Rodríguez Ricard**

*Turing-Hopf Patterns near the onset*

Diffusion-driven instabilities in reaction diffusion systems generated by the limit cycle which appears due to a Hopf bifurcation are considered. Conditions under which the limit cycle destabilizes are weaker than the conditions destabilizing the steady state, for instance it is not necessary that the diffusion coefficients be different enough. Twinkling patterns are to be expected provided close enough, or even equal, diffusion coefficients. Finally, we consider the wave initiation of twinkling pattern via a travelling wave of change of phase type for the cycle amplitude.

**Karl-Theodor Sturm**

*Optimal Transport from Lebesgue to Poisson*

We study couplings  $q^\bullet$  of the Lebesgue measure and the Poisson point process  $\mu^\bullet$ , i.e. measure-valued random variables  $\omega \mapsto q^\omega$  s.t. for a.e.  $\omega$  the measure  $q^\omega$  on  $\mathbb{R}^d \times \mathbb{R}^d$  is a coupling of  $\mathfrak{L}^d$  and  $\mu^\omega$ . For any given  $p \in (0, \infty)$  we ask for a minimizer of the mean  $L^p$ -transportation cost

$$\mathfrak{C}(q^\bullet) = \sup_{B \subset \mathbb{R}^d} \frac{1}{\mathfrak{L}^d(B)} \mathbb{E} \left[ \int_{\mathbb{R}^d \times B} |x - y|^p dq^\bullet(x, y) \right].$$

The minimal mean  $L^p$ -transportation cost turns out to be finite for all  $p$  provided  $d \geq 3$ . If  $d \leq 2$  then it is finite if and only if  $p < d/2$ .

Moreover, in any of these cases we prove that there exist a unique translation invariant coupling which minimizes the mean  $L^p$ -transportation cost. In the case  $p = 2$ , this 'optimal coupling' induces a random tiling of  $\mathbb{R}^d$  by convex polytopes of volume 1.

