MSM3P22/MSM4P22
Further Complex Variable Theory & General Topology

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Banner Code: 06 22792 (MSM3P22), 06 22793 (MSM4P22)
Credit Value: 20 credits **Level:** H (MSM3P22), M (MSM4P22)
Prerequisites: MSM2B (or MSM3G03) and MSM2O3 (or MSM3P04)
Co-requisites: None

You cannot combine:
MSM3P22 with MSM3P09, MSM3P10, MSM4P22, MSM4P09, MSM4P10.
MSM4P22 with MSM3P09, MSM3P10, MSM3P22, MSM4P09, MSM4P10.

Description

This module continues the study of complex-valued functions of a complex variable begun in MSM2B (06 22497). Analytic functions have more structure that their real counterparts, the differentiable functions, and this extra structure results in a fascinating theory and widespread applications in other areas of mathematics and beyond. The course touches on both these aspects: conformal maps, which have many applications, are discussed, and the techniques of contour integration are extended to deal with functions involving logarithms and roots, while the theory and structure of analytic functions are used to give simple and elegant proofs of the fundamental theorem of algebra. This module also introduces metric and topological spaces as abstract settings for the study of analytical concepts such as convergence and continuity. This generalization allows one, for example, to regard functions as points of a space and to consider various ways in which the function can be the limit of other functions. Extra structure is introduced: compactness, for instance, is shown to be the proper generalization of the closed bounded intervals often used in analysis on the real line. The course may end with an application: for example, using topological techniques, one can prove the existence of continuous functions on the real line which fail to be differentiable at any point.

Delivery: 44 hrs of lectures and 10 hours of examples classes.

Assessment: 90% on one three-hour examination in the Summer Term, 10% based on work during term-time.

Learning Objectives, Outcomes

Skills you should have acquired and things you should be able to do by the end of the module are detailed below for each part:

Further complex variable theory: Better understand complex valued functions of a complex variable; understand conformal mappings, in particular Möbius transformations, and apply them in particular examples; evaluate a wider range of contour integrals, in particular integrals around branch points; understand and apply various results in the theory of analytic functions such as the principle of the argument.

General topology: Understand the definitions of metric and topological spaces and verify these definitions in examples; manipulate open and closed sets; understand the definitions of compactness and connectedness and prove basic results involving these properties.

For students taking MSM4P22 only: By the end of this module, students should also be able to explore these topics beyond the taught syllabus.

Information specific to MSM3P22/MSMS4P22 General Topology

Syllabus

- Set theory: infinity, arbitrary unions and intersections, the power set, de Morgan's Laws, the Axiom of Choice, cardinalities, filters and ultrafilters.
- Topology I: the notion of a topology on a set; open and closed sets, continuity; separation axioms; describing topologies, bases; limit points, interiors and closures; examples
- Topology II: subspaces; products; quotients; examples
- Separation: the separation axioms, T_1–T_4;
- Compactness: compact, Hausdorff spaces; compactness vs closed subsets; preservation under continuity; the Tychonoff Product Theorem; examples
- Connectedness: connectedness; path connectedness; local connectedness; simply connected spaces; examples
- Something interesting taken from: continua, space filling curves; the fundamental group; dimension theory; continua; hairy balls; Jordan Curve Theorem; metrization theory; Baire Category Theorem; Urysohn's Lemma and the Tietze Extension Theorem; anything else related to applications of topology in other areas.

Recommended texts

The lectures and handouts should provide you with all the information you need to complete the module, but you are encouraged to use any other sources of information that you may find helpful. There are many excellent textbooks covering this material and you can find lots of them in the main library; look for introductions to topology or books on general or analytic topology. Algebraic and differential topology are different topics but many books on algebraic topology will have an good introduction to general topology. The perspective these books take can vary so you should shop around until you find one that suits you.

- *Introduction to Metric and Topological Spaces* by W. A. Sutherland, published by OUP. This book contains most of the theorems that we will cover, it is a well-written and clear text, though is a little dry and does not contain all of the examples we will look at.
- *Introduction to Topology* by Bert Mendelson, published by Dover Books. Cheap and a classic.
- *Topology: a first course* by James R. Munkres, Prentice-Hall, Inc. A very popular topology text.
- *Topology* by John G. Hocking and Gail S. Young, Addison-Wesley Publishing Co. A great book especially if you go on to do some algebraic topology. Probably leaves more for the reader to do than Sutherland.
- *General topology* by Stephen Willard, originally Addison-Wesley, reprinted by Dover. Classic, well-written, very comprehensive with a fantastic index.

- *General topology* by Ryszard Engelking, second edition, Heldermann Verlag. Fantastic, encyclopedic text, beautifully written and organized. Would definitely take you to graduate level and beyond.
- *General topology* by John L. Kelley, originally van Nostrand, reprinted by Springer-Verlag. Another classic along the lines of Engelking.

Lectures, examples classes and handouts

There are two lectures each week: one on Tuesdays at 14:00 and one on Fridays at 16:00. Both take place at Watson 310.

Lectures will consist mainly of explanations on the blackboard, and questions and comments are very welcome during them. Handouts containing a summary of things covered in class will be provided. Spare copies will be placed in the red tray next to the pigeon holes, and you will be able to download them from my webpage.

The examples class will be held every two weeks on Tuesdays at 13:00, starting in Week 3 of term. They also take place at Watson 310.

Assessment

Continuous assessment: A total of 5% of the module mark is available from course work on the General Topology part during term (the other 5% of the continuous assessment mark will come from the Further Complex Variable Theory part.)

In order to obtain this 5% you should submit solutions to the problem sheets issued throughout the term. You are free to choose which problems you answer. Each problem is usually worth 1 mark, though there may be some worth more (as indicated in problem sheets.) Solutions to problems will be marked either right or wrong, and you may re-submit solutions as many times as you like until they are marked correct or we have reached the end of term. You are free to hand in solutions to as many questions as you want. There are no time limits to handing in work provided you hand it in during the term.

The continuous assessment mark for 3P22a General Topology will be calculated as follows: There will be 5 assessment marks, CA1-CA5, recorded on the university database. The best 4 of these will count towards your continuous assessment mark for this half of the module, so that each one is worth 1.25% of total the module mark. Each CA mark will be out of 3. To gain 3 marks on a Continuous Assessment mark you will need correct answers to 3 problems. For example if you get just 3 correct answers during the term you will get 3/3 for CA1 and 0 for CA2-CA5. If you get 11 correct answers you will get 3/3 for CA1, CA2, and CA3, 2/3 for CA4 and 0/3 for CA5. Getting another question right would then get you 3/3 for CA4 as well.

The Examination for MSM3P10/4P10: The examination is worth 90% of the module mark, and is a three-hour examination to take place in June. In it you will have to correctly answer 4 questions from 6 to get full marks.

MSM3P22/4P22 is a pure module. This means that there will be a number of formal definitions, theorems and proofs. It is important that you know the statements of any definitions and theorems. The examination may well ask you to state definitions and to state and prove theorems covered in the course. However, don't try to learn proofs by heart. It is *much easier* to remember the key steps (use the outline proofs) and work out the rest as you go.

Contacting the lecturer:

My office is room 215 of the Watson Building, my telephone extension is 46586 and my email address is `j.a.canizo@bham.ac.uk`. My Office Hours will be published on my door, but you are welcome to call or write to me in order to arrange a different time to meet. You may also try to drop by my office at any time and if possible we can talk right away.